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Active cancellation of pure tones in an excited jet

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This paper describes the results of experiments conducted on a circular jet simultaneously excited by two different acoustic tones. By varying the phase between two signals at harmonically related frequencies, control can be exercised on the process of harmonic generation – sometimes the process being virtually destroyed. This is shown to be so for both harmonic and subharmonic generation, but the latter is more difficult to control.

1. Introduction

Acoustic excitation has been widely used to analyse the structure and development of turbulent flows. The circular jet with controlled excitation was first studied by Crow & Champagne (1971). Chan (1974), Petersen, Kaplan & Laufer (1974), Browand & Laufer (1975), Bechert & Pfizenmaier (1975), Moore (1977), Zaman & Hussain (1980), Reynolds & Bouchard (1981) have all developed and expanded on that theme. Crow & Champagne found a 'preferred' frequency for which the jet was most sensitive. Chan measured the spatial development of pressure waves. Petersen *et al.* observed that jet excitation brought about a suppression of the turbulence intensity. But a large amplification of the broadband jet noise was found by Bechert & Pfizenmaier and independently by Moore. Zaman & Hussain studied extensively two modes of vortex pairing in a circular jet flow, while Reynolds & Bouchard (1981) modified the pairing by exciting the jet at a particular frequency.

Milling (1981), and Liepmann, Brown & Nosenchuck (1982), have conducted experiments in which Tollmien–Schlichting disturbances developing in a boundary layer on a flat plate were nearly cancelled by interference with a second wave which was arranged to be 180° out of phase with the original disturbance. Ho & Zhang (1981), and Ho & Huang (1982) have demonstrated how vortex merging in a mixing layer can be significantly modified by periodic forcing. In a similar way Reynolds & Bouchard (1981) have regulated the ring-vortex structure in the mixing layer of a round jet by exciting the jet with periodic axial disturbances of high amplitude (up to 30 %).

Though there has been this large scale of activity and interest in the effects of external excitation of turbulance we are not aware of many experimants in which two different excitation frequencies are used simultaneously. Ho & Zhang's (1981) experiments on a two-dimensional shear layer did involve multiple-frequency excitation, but not in as controlled a manner as we report below.

Ronneberger & Ackermann (1979) also excited a jet at two different frequencies in an experiment similar to ours, but they do not report on the variation of the response as the relative phase angle of the harmonic excitation is changed. That is

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the aspect that we have found most interesting and which is the central theme of this paper. Again this is an aspect that was not covered in Miksad's (1973) experiment where the shear layer downstream of a splitter plate was artificially excited. This problem might be important, since in industrial conditions jets are most often excited by a multiple discrete-component spectrum. The purpose of this paper is to present the results of an experiment in which the control of pure tones developing in an excited jet is achieved by excitation with sound at two related but different frequencies.

We encountered the phenomenon described in this paper while investigating the possibility of maintaining the stability of the initial jet shear layer by active control with secondary sources. That prospect rests on the notion that waves can be superposed without mutual interference – an essentially linear view of shear-layer disturbances. But we found that there are in the jet some effects whose origin lie in definite nonlinear processes, yet they too can be superposed to interfere (sometimes destructively) by wave superposition, and the way they interact is not noticeably different from linear behaviour. We thought that to be interesting enough to report here, though we recognize that the underlying physics of the large-eddy process is much too complicated to admit anything more than a qualitative explanation, and we give our view of such an explanation at the end of the paper.

2. Experimental conditions

The jet rig consisted of a stagnation chamber and a 2.54 cm diameter nozzle with a contraction (area) ratio of 12.8 (figure 1). It was powered by a fan that gave operating jet speeds up to 20 m/s. The measurements reported here were performed at an exit velocity of 10.32 m/s, which corresponds to a Reynolds number of 1.75×10^4. At this speed the exit plane turbulence level was measured to be 0.2 %.

The excitation source employed was similar to that used by Kibens (1979). An azimuthally coherent perturbation was introduced locally at the nozzle exit through a thin slit surrounding the nozzle. The excitation was driven with 4 loudspeakers located in the exciter chamber (figure 1). The width of the acoustic-driver exit slit was 3 mm.

The excitation signal supplied to the loudspeakers, which was measured to have linear amplitude and phase response in the experimental range, was obtained in the following way. A phase-lag generator provided two signals, $A_1(\omega t)$ and $A_2(2\omega t+\phi)$, the frequency of one of the signals being twice that of the other. These two signals were added to obtain $A_1(\omega t)+A_2(2\omega t+\phi)$. In most of the results presented here the surging amplitude, as measured at the centre of the nozzle exit, was an axial-velocity variation of 2 % of the mean jet speed, and this level was continually monitored and controlled in our experiments.

The flow field was measured with hot-wire anemometers, and the near-field pressure measurements were made outside the jet with a condenser microphone. The microphone was positioned near the edge of the flow, at the downstream position where the activity of the jet was a maximum when excited at its preferred frequency, a Strouhal number of 0.3.

3. Experimental results

3.1. *Near-field pressure*

The near-field spectrum was measured for a forcing amplitude that provided a surging velocity at the nozzle exit, which was measured to be 2 %. This level was held constant

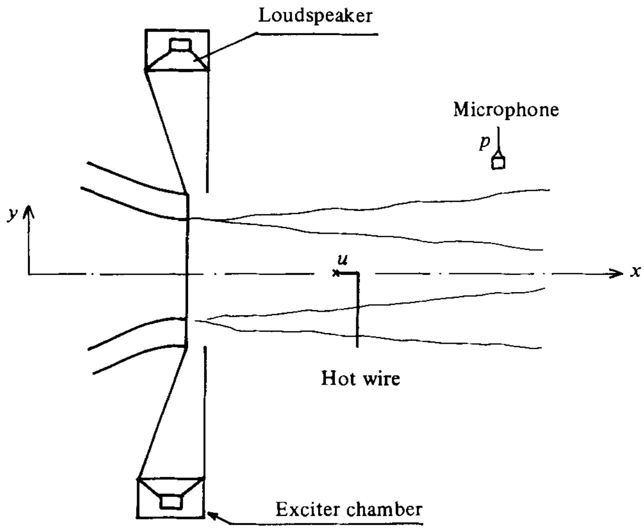

FIGURE 1. A schematic illustration of the jet nozzle surrounded by an annular ring connecting the nozzle exit to the chamber containing four loudspeakers.

at the various excitation frequencies which ranged from a Strouhal number f_e^*, based on the mean jet velocity and nozzle diameter, from 0.15 up to 2. Two different domains can be distinguished: the low-frequency range ($f_e^* < 0.45$) where the pressure spectrum is dominated by the excitation frequency and its harmonics (figures 2*a, b*), and the high-frequency range ($f_e^* > 0.45$) where only subharmonics appear (figure 2*d*). The response of the jet to forcing at $f_e^* = 0.45$ was found to be very weak, a feature that has been observed theoretically by Acton (1980). Harmonics are produced by the nonlinear evolution of the fundamental instability wave (Crow & Champagne 1971; Crighton & Gaster 1976; Crighton 1981). Subharmonics, however, are not so much generated but encouraged to grow by the interaction of the excited waves with other naturally occurring disturbances in the flow, disturbances that already have a component at the subharmonic frequency; these are associated to the jet column vortex-pairing mechanism (Zaman & Hussain 1980).

Owing to the existence of two different response modes produced by two different nonlinear mechanisms, it seemed to us that their interaction might be interesting. To study this interaction the jet was first excited at low frequency ($f_e^* = 0.3$), and the cancellation of the harmonic attempted by superposing a secondary excitation. In a second experiment high-frequency excitation was used ($f_e^* = 0.9$) and the control of the subharmonic was achieved with secondary excitation at half the frequency, i.e. 0.45.

3.2. *Cancellation of a harmonic*

The jet was excited at its preferred frequency ($f_e^* = 0.30$) to produce a surging level of 2 %. The near-field pressure spectrum E_p and the excitation signal $A(t)$ are plotted in figures 3(*a, b*). As already seen, this spectrum exhibits two peaks, one at the excitation frequency and another at the harmonic $2f_e^* = 0.60$. In order to control the harmonic amplitude, a second signal of frequency $2f_e^*$ and amplitude $A_2(2f_e^*)$ is added to the initial signal $A_1(f_e^*)$ and is applied to the jet. The phase difference between A_2 and A_1 can be varied.

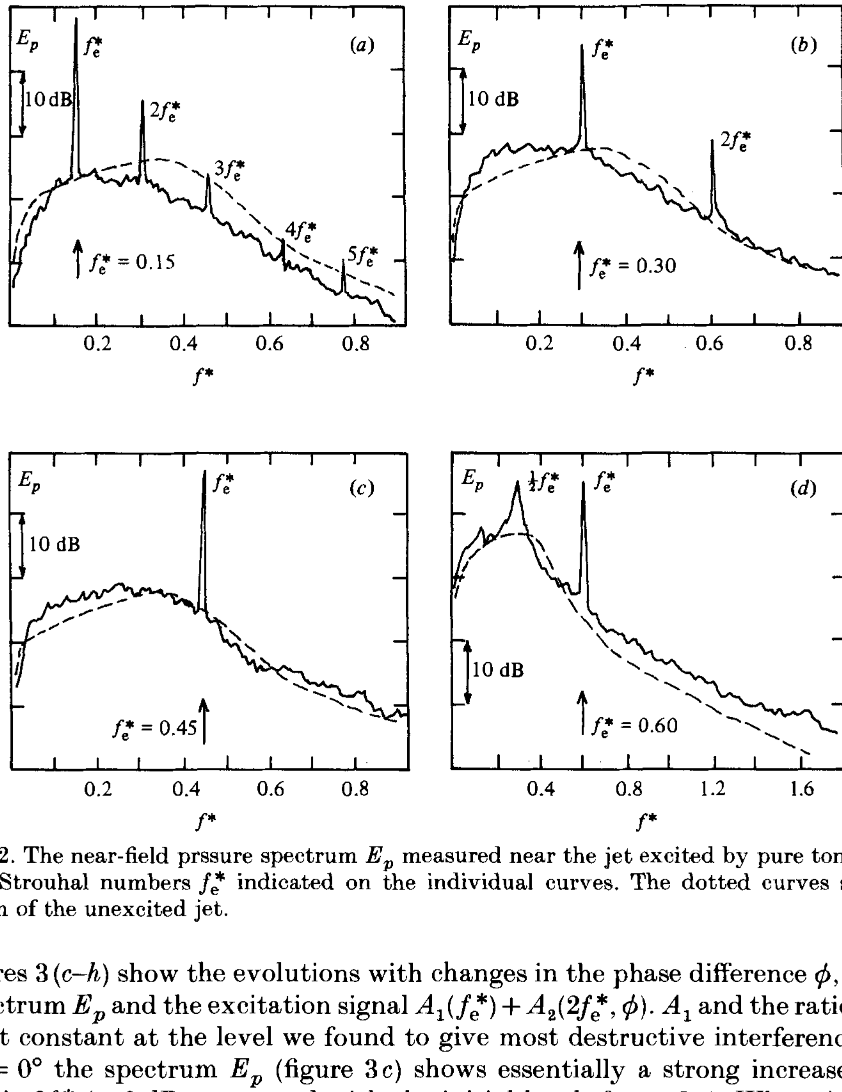

FIGURE 2. The near-field prssure spectrum E_p measured near the jet excited by pure tones at the various Strouhal numbers f_e^* indicated on the individual curves. The dotted curves show the spectrum of the unexcited jet.

Figures 3 (*c*–*h*) show the evolutions with changes in the phase difference ϕ, of both the spectrum E_p and the excitation signal $A_1(f_e^*)+A_2(2f_e^*, \phi)$. A_1 and the ratio A_2/A_1 are kept constant at the level we found to give most destructive interference, 0.38. For $\phi = 0°$ the spectrum E_p (figure 3*c*) shows essentially a strong increase of the harmonic $2f_e^*$ (+6 dB, compared with the initial level, figure 3*a*). When ϕ reaches 50°, the amplitude of $2f_e^*$ is nearly constant, but that of the fundamental decreases (−4 dB). Finally, for $\phi = 180°$ the opposite result is obtained: there is an amplification of the fundamental (+5 dB) and a cancellation of the harmonic $2f_e^*$.

The streamwise evolutions of the fluctuating flow velocities u_{f^*}, u_{2f^*} corresponding to the two frequencies f_e^*, $2f_e^*$ is shown in figure 4, for the preceding excitation conditions. When the signal A_1 alone is applied, the streamwise evolutions of u_{f^*} and u_{2f^*} are similar to those obtained by Crow & Champagne (1971). The upstream drift of the maximum-amplitude point is very slight, and might be due to an effect of the Reynolds number ($Re = 10.6 \times 10^4$ in the experiments of Crow & Champagne). For the $A_1 + A_2$ signal and a phase difference $\phi = 0$, the downstream evolution of u_{f^*} is modified. Although its rate of growth seems to remain constant, its maximum-amplitude point moves downstream to the axial position $x/D = 3.5$. At the opposite extreme, the maximum point for u_{2f^*} moves upstream and covers a broad spread from $x/D = 1.5$ up to $x/D = 3$. When the phase difference reaches $\phi = 180°$, the rate of growth of u_{f^*} is strongly increased, but the maximum-amplitude point stays at

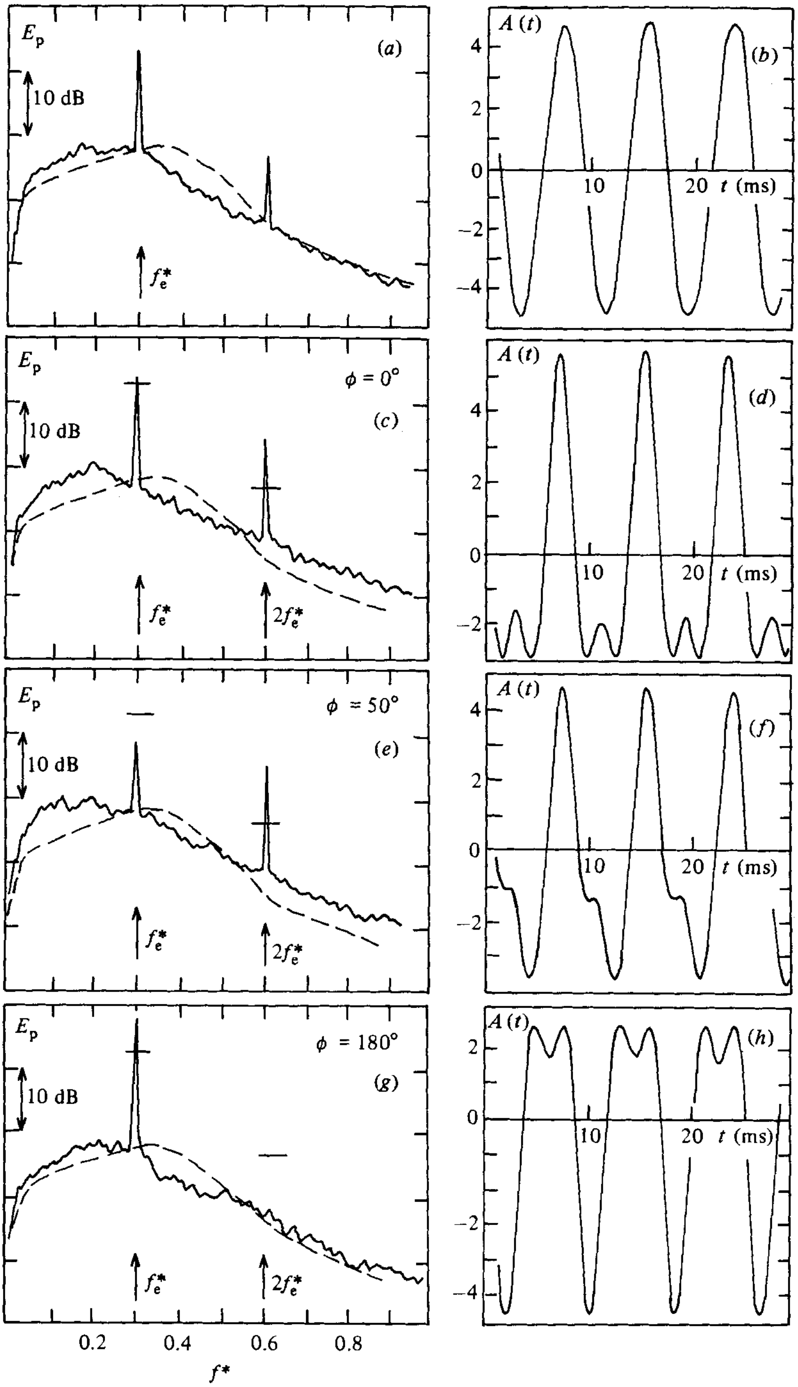

FIGURE 3. A series of near-field pressure spectra measured when the jet is excited at two frequencies, Strouhal number f_e^* and $2f_e^*$. The phase angle is indicated on the individual curves and the time history of the excitation signal $A(t)$ is also shown. ($A(t)$ in arbitrary units.)

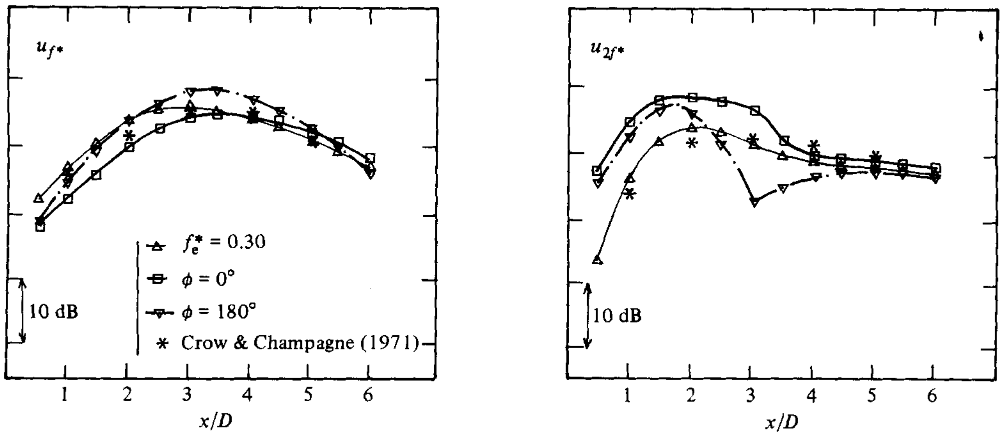

FIGURE 4. The streamwise evolution of the velocity components at Strouhal numbers $f_e^* = 0.3$ and $2f_e^* = 0.6$. These measurements were made on the jet axis.

$x/D = 3.5$. A most interesting effect is seen for u_{2f*}; the amplitude of this component grows until $x/D = 1.5$, then decreases sharply (more than 20 dB at $x/D = 3$), and finally increases slightly from $x/D = 3$ up to $x/D = 4.5$. Our survey on the conditions that affect the cancellation of the harmonic showed an influence of both the amplitude ratio A_2/A_1 and the excitation level, but we could not detect any trend significant enough to merit any more detailed account being given here.

3.3. *Cancellation of a subharmonic*

The jet was then excited at the frequency $f_e^* = 0.90$ and a surging level of 2 %. The response spectrum is dominated by two peaks, one at the excitation frequency f_e^*, and another at the subharmonic $\frac{1}{2}f_e^*$, of amplitude 9 dB greater than that of f_e^*. Once again, to control the subharmonic amplitude, a second signal of frequency $\frac{1}{2}f_e^*$ and amplitude $A_3(\frac{1}{2}f_e^*)$ was added to $A_1(f_e^*)$, and applied to the jet.

Figures 5(*a–f*) show the evolution, with the phase difference ϕ between A_3 and A_1, of both the near-field pressure spectrum E_p and the excitation signal $A_1(f_e^*)+A_3(\frac{1}{2}f_e^*, \phi)$. The ratio $A_3/A_1 = 0.38$ is kept constant as well as the excitation level.

Again the ratio of the two excitation signals is maintained at the value we found to give the most complete destructive interference, and this happens to be at a value equal to that found most effective when cancelling the harmonic wave described above. If there is a fundamental reason why this should be so, we don't know of it. Without the Strouhal number 0.9 forcing, the jet responds little to excitation at Strouhal number 0.45, as we have already indicated. But the subharmonic generated at Strouhal number 0.45 by 0.9 forcing is much more susceptible to control and the condition that controls it most effectively is when the amplitude of the subharmonic excitation is 0.38 that of the fundamental excitation level, just as was the case for harmonic generation. For $\phi = 0°$, the spectrum E_p is slightly modified by the additional excitation, there is a small decrease of the f_e^* component, the subharmonic $\frac{1}{2}f_e^*$ is unchanged and a second subharmonic $\frac{1}{4}f_e^*$ appears. When ϕ is increased to 45°, the $\frac{1}{4}f_e^*$ component vanishes and the amplitude at $\frac{1}{2}f_e^*$ is strongly reduced (-9 dB); its complete cancellation was never obtained.

The streamwise evolutions of the fluctuating flow velocities u_{f*}, $u_{\frac{1}{2}f*}$ corresponding to the two frequencies f_e^*, $\frac{1}{2}f_e^*$ are shown in figure 6 for the same excitation conditions.

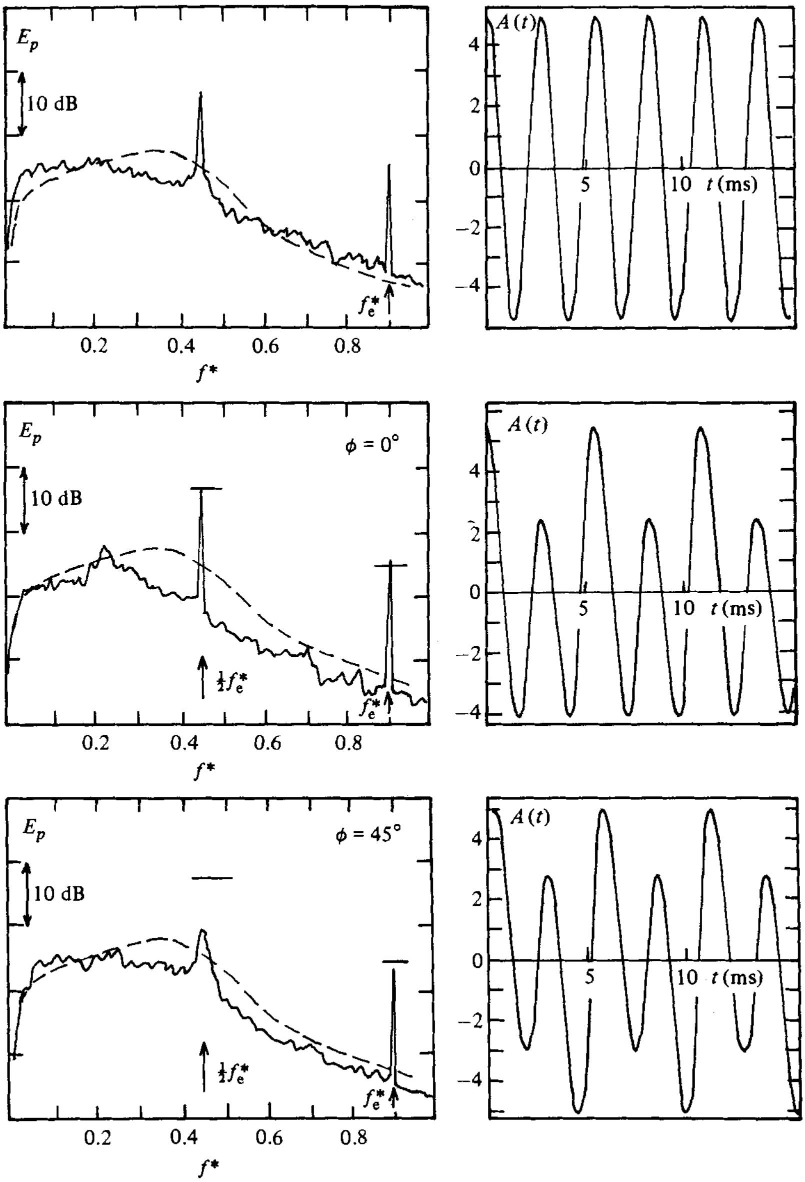

FIGURE 5. The near-field spectra and excitation signals for a jet excited at a Strouhal number 0.9. ($A(t)$ in arbitrary units.)

When the signal A_1 alone is applied, our results for the streamwise evolutions of u_{f^*} and $u_{\frac{1}{2}f^*}$ are similar to those obtained by Zaman & Hussain (1980). For the $A_1 + A_2$ signal and a phase difference $\phi = 0$, the E_p spectrum is slightly modified, the amplification of u_{f^*} is smaller, but that of $u_{\frac{1}{2}f^*}$ is the same as before. When the phase difference ϕ increases up to 45°, the E_p spectrum is strongly modified. The same kind of behaviour was observed when we studied the streamwise evolution at frequencies $\frac{1}{4}f_e^*$, $\frac{3}{2}f_e^*$ and $2f_e^*$. Our results on the modification to the growth rate of u_{f^*} and $u_{\frac{1}{2}f^*}$ when the phase varies are consistent with those obtained earlier by Ho & Zhang (1981) for a two-dimensional shear layer.

4. Tentative physical interpretation

4.1. *Harmonic cancellation*

Figure 2 shows that the amplitude of the harmonic $2f_e^*$ is always about 15 dB lower than that at the excitation frequency f_e^*, so the process of harmonic generation seems to be only weakly nonlinear (Crighton & Gaster 1976), and we can expect that different wave elements might be superposed using a principle of 'nearly linear' approximate superposition. It is possible to seek a qualitative explanation by analogy with the known behaviour of unstable laminar shear layers. Stuart (1960) and Schade (1964) express the perturbation on a weakly unstable laminar shear layer in the form:

$$\psi = \psi_0 + A\psi_1 \exp(\mathrm{i}\alpha(x-ct) + A^2\psi_2 \exp(2\mathrm{i}\alpha(x-ct)) + \text{complex conjugates} + \dots \quad (1)$$

α is the wavenumber of the initial perturbation, c the phase velocity; ψ_1 and ψ_2 are solutions of the Orr–Sommerfield equation. According to Betchov & Criminale (1967), one can expect that the various components will be synchronized by the nonlinear process.

If such a decomposition is used for the waves induced in our turbulent jet, it is then easy to rationalize the cancellation of the harmonic at $2f_e^*$ in the following way. A first disturbance $A_1(f_e^*)$ is applied to the jet whose response is of the type described by (1). If a second disturbance, say $A_2(2f_e^*, \phi)$ with frequency $2f_e^*$ but with the same phase as A_1, is applied, there is an amplification of the harmonic, due to the superposition. That is what is observed and illustrated in figures 3 (*c*) and 4. But, when the phase difference ϕ between A_2 and A_1 is increased by 180°, superposition implies the cancellation of the harmonic at $2f_e^*$, and that is the condition corresponding to figure 3 (*g*).

4.2. *Subharmonic cancellation*

The cancellation of the subharmonic is a much more subtle affair, for which the 'subharmonic-resonance' theory of Kelly (1967) might be relevant, though in his theory the subharmonic amplitude was much smaller than the primary tone; this is not the case in our experiments (figure 5). Support for the view that subharmonics play an important role in vortex-interaction processes comes from experiments (Ho & Huang 1982; Laufer & Zhang 1983), as well as from numerical computations

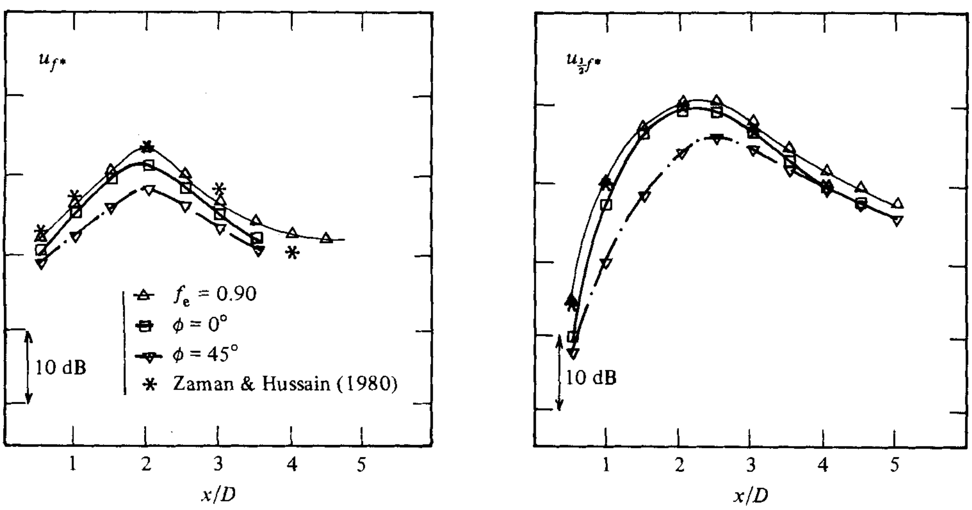

FIGURE 6. The development of the axial-velocity fluctuation on the jet axis at Strouhal numbers 0.9 and 0.45, for a jet excited at Strouhal number 0.9.

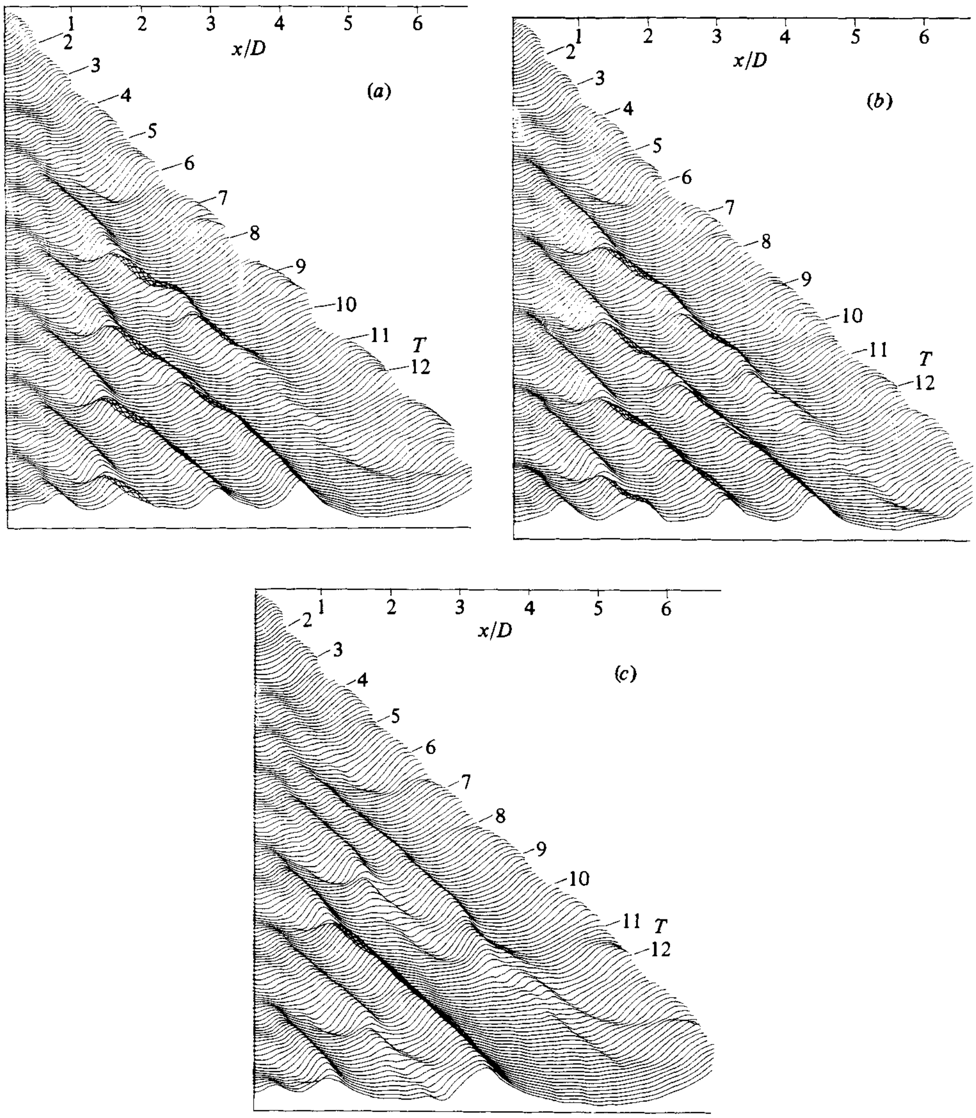

FIGURE 7. Acton's calculated evolution of the eddy structure in a round jet perturbed by two different frequencies of amplitudes A_1 and A_2 corresponding to conditions of figure 5. (*a*) $A_1 = 0.1$; (*b*) $A_1 = 0.1$, $A_2 = 0.0375$, $\phi = 0°$; (*c*) $A_1 = 0.1$, $A_2 = 0.0375$, $\phi = 45°$. Each curve is the axial distribution of axial velocity at equal time intervals.

(Patnaik, Sherman & Corcos 1976). That is where we think the explanation for the effects we have observed is to be found. Ho & Huang show that the subharmonic amplification is a catalyst for vortex pairing. Patnaik *et al.* simulated a two-dimensional vortex pair interacting with a subharmonic wave, and found that there are certain phase differences between the motion of the vortex pair and the subharmonic which effectively amplify or inhibit vortex coalescence. They concluded that the kinematics of this subharmonic interaction depend dramatically on the phase relationship in which the two waves are initially superposed. That is precisely the situation we have observed in our experiments, the results of which are shown in figures 5 and 6. When a subharmonic disturbance $A_3(\frac{1}{2}f_e^*)$ with the same phase as the fundamental perturbation $A_1(f_e^*)$ is added to A_2 and applied to the jet, there is a slight amplification of the subharmonic $\frac{1}{2}f_e^*$. But when the phase $\phi = 45°$ a cancellation of that subharmonic is observed.

Those results are also consistent with Acton's (1980) modelling of the eddies in an axisymmetric jet. She has developed for us her original model, which represented the jet shear layer by the superposition of vortex-ring elements, to take into account the two excitation frequencies. The axial distribution of axial and radial velocities, at equal time intervals, is then obtained for various excitation conditions. Figures 7 (*a*, *b*) show that the same downstream evolution of the jet characteristics is obtained using a single excitation frequency ($f_e^* = 0.9$) or two frequencies with the same phase ($f_e^* = 0.9$ and $\frac{1}{2}f_e^* = 0.45$). However, when the phase difference $\phi = 45°$, the downstream evolution of the jet is very strongly modified (figure 7*c*), and the evident decrease in the regularity of the subharmonic is, we think, consistent with our experiment.

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